STUDIES ON NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS USING GENERALIZED HOMOTOPY ANALYSIS METHOD AND TRANSFORM TECHNIQUES

ABSTRACT

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ABSTRACT

The computational methods for solving linear and nonlinear fractional differential equations are of essential requirement to analyze various categories of science and engineering problems. Hence, many researchers have developed several mathematical methods to solve the linear and nonlinear fractional differential equations. This dissertation presents new hybrid techniques that are based on the integral transform and Homotopy Analysis Method (HAM) to solve the fractional differential equations and to study the importance of the computational methods through the analysis of solving widely used fractional differential equations.

A hybrid computational method 'Generalized Homotopy Analysis Method (GHAM)' based on G-transform and HAM is proposed and applied to solve various categories of linear and nonlinear differential equations. Using GHAM, the closed form series solutions are obtained for the higher-order nonlinear ordinary and partial differential equations and the computational efficiency of the method is studied. Compared to HAM, GHAM completely avoids multiple differentiation and integration in solving higher-order nonlinear differential equations.

Due to the possession of the non-local property of the fractional derivative, the notion of Caputo type fractional derivatives are introduced in G-transform and a novel technique 'Fractional Generalized Homotopy Analysis Method (FGHAM)' is proposed using Mittag-Leffler function and its standard properties are studied. The performance of the proposed method is analyzed by solving various categories of nonlinear fractional differential equations like Navier Stokes's model and Riccatti equations. The closed form series solutions obtained using FGHAM show an excellent agreement with the other existing methods and the results are statistically validated.

Further, FGHAM is applied to solve the fractional Black-Scholes model in financial mathematics. The convergence of the series solutions is well established using Cauchy criteria. Using the numerical example, analytical expression for the implied volatility is derived and the non-local behavior is studied for the various values of the fractional parameter and the results are statistically validated. The solutions obtained using FGHAM are compared with the other existing methods including Monte-Carlo simulation and observed to have a good agreement. The implementation of FGHAM overcomes the difficulty of tackling iterative differentiation and integration in solving the fractional nonlinear differential equations.

Moreover, due to the importance of the various types of fractional derivatives, the notion of modified Riemann-Liouville derivatives are introduced in G-transform using Mittag-Leffler function. Another new hybrid technique Fractional Generalized Homotopy Analysis Method using Modified Riemann-Liouville derivative (MRFGHAM) is proposed and its standard properties are studied. In order to study computational efficiency, the proposed method is implemented to analyze a non-fatal disease epidemic model. The spreading process of a non-fatal disease is studied for the various values of the fractional parameter with the statistical validation. The comparative study shows an excellent agreement with the other existing methods.