# STUDIES ON SOME ALGEBRAIC STRUCTURES IN ABSTRACT CELLULAR COMPLEX

**A THESIS**

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# ABSTRACT

Topology, a well-known area of Mathematics, is a study of relationship between two spaces, particularly the geometric structures with their properties, deformations, and mapping between them. It has recently become an important area of applied mathematics. The study of basis is useful to define the topological space of a set $X$, from the smaller collection of a subset $X$. Each point $x$ in $X$ is enclosed in a family of subsets of topological space $X$, called its neighbourhoods.

The studies of connectedness and path connectedness are useful to characterize the different topological spaces. For example, removing a point from the plane $R \times R$ leaves a connected space remaining whereas removing a point from the real line $R$ does not. It is insufficient in the case of torus and sphere while using usual invariants of topology. Further, identifying similar topological spaces using homeomorphism is a challenging task for topologists. Hence, topologists introduced the concept of fundamental groups in topological spaces by initiating the notions of homotopy in topological spaces and by establishing that the two topological spaces are homeomorphic if their corresponding fundamental groups are isomorphic to each other. Because homotopy is concerned about the classification of geometric regions by studying the paths which are drawn in the region. Moreover, homotopy gives information about the basic structure or holes of the space.

Many topological properties like connected, neighbourhood, closure, interior, boundary, etc., are used in digital image analysis. Hence, researchers studied the aforementioned properties including algebraic topological properties in digital images and have developed the algorithms for digital image analysis

using the digital topological properties. For this, initially Rosenfeld introduced the concept of Digital topology and represented the digital image as a neighborhood graph, in which vertices were represented as pixels and edges as adjacent pixels. Though the representation of the digital image as a neighbourhood graph was useful in digital image analysis, the neighbourhood properties of digital image inherit the boundary and connectivity paradoxes. Kovalevsky introduced the axiomatic approach with the lower-dimensional cells to overcome the connectivity and boundary paradoxes of the digital image for getting the perfect boundary from the digital image. Krishnan and Vijaya implemented the Chain Code algorithm which was developed by Kovalevsky in the various images in the standard grid and the algorithm showed better results; however, the lower-dimensional cells were not discussed.

The notion of Abstract Cellular Complex deals with the topological properties of lower-dimensional space which is important in the study of the digital image in a combinatorial grid. This motivates to initiate the notion of algebraic invariants in the Abstract Cellular Complex, to study and to characterize the digital images through the Abstract Cellular Complex. The implementation of invariants of algebraic topology such as homotopy and the fundamental group in the Abstract Cellular Complex give the ingress to identify the different digital images and to extract their features in digital image processing techniques.

This dissertation primarily focuses on the algebraic structures in the Abstract Cellular Complex. The following topics are studied and some interesting results are obtained: basis and subspace topology in the Abstract Cellular Complex, product and connected complexes in the Abstract Cellular Complex, mappings on the Abstract Cellular Complex for conversion of standard grid to combinatorial grid and vice-versa. Further, image analysis based on Abstract Cellular Complex through the Modified Chain Code algorithm which is based on defined mappings on the Abstract Cellular Complex is proposed and the proposed Modified Chain Code algorithm is compared with the other algorithms such as Simple Boundary Follower, Modified Simple Boundary Follower, Moore-neighbourhood Tracing, and Theo Pavlidis Algorithm, Kovalevsky's Chain Code Algorithm. Moreover, the concepts of cellular homotopy, cellular path homotopy, cellular fundamental group, abstract cellular simplicial homology, cellular fixed point property, cellular homotopy fixed point property, winding number, abstract cellular simplicial map and abstract cellular simplicial approximation of the abstract cellular complex were introduced and some of their basic properties were investigated.

Chapter 2 introduces the notions of basis and subspace topology in Abstract Cellular Complex and characterizes by using the lower-dimensional cells. The definition of conversion and inversion mappings are initiated for the conversion of a given digital image from the standard grid to the combinatorial grid and vice versa. Further, this chapter investigates the notions of product and connected complexes in the abstract cellular complex and studies some of their basic properties.

Chapter 3 develops the Modified Chain Code Algorithm and identifies the boundary pixels of a digital image along with their corresponding lower-dimensional cells that are explicitly obtained. The developed MCC algorithm is implemented for the various medical images. The performance of the proposed MCC algorithm is analyzed by using the metrics like Peak Signal Noise Ratio (PSNR) and Hausdorff Distance (HD). The experimental results reveal that this proposed algorithm can be effectively used as a boundary tracing algorithm for the various digital images.

Chapter 4 initiates the notions of cellular homotopy, cellular path homotopy through the continuous connected preserving map and investigates some of their fundamental properties. Further, the cellular homotopic relation among the maps forms an equivalence relation by using the cellular homotopic relation between the maps and it generates an equivalence class among cellular homotopic maps (cellular path homotopic). Based on a binary operation on the class of cellular path homotopic maps, the notion of the cellular fundamental group is initiated in abstract cellular complex and investigates some of their basic properties. Finally, the concepts of homology and fixed point property in the connected abstract cellular complex are discussed.

Chapter 5 initiates the notion of the winding number and discusses the computation of the winding number of the digital image in the abstract cellular complex. Further, it introduces the concept of free homotopy on the abstract cellular complex between the loops and studies some of their properties through the winding number of the loop with respect to a 0-dimensional cell. Moreover, this chapter discusses the concepts of the subdivision through the barycentric coordinates of the abstract cellular complex. Finally, it also discusses the notions of a subdivision of the abstract cellular complex and abstract cellular simplicial approximation.