NON-MARKOVIAN BULK QUEUEING MODELS WITH EFFECTIVE UTILISATION OF IDLE TIME

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Queueing theory deals with the study of waiting lines. A systematic study of the queueing system provides a knowledge base that can be applied to monitor and improve the efficiency of many queueing system in the real world. The study of queueing systems has attracted the attention of researchers, since it has wide range of applications in the fields of manufacturing, transport systems, communication systems and local area networks.

Queueing systems with bulk arrival and bulk service with the possibility of server failures and the effective utilization of server idle time by allocating the server to some other work(vacation models) are often important in real life situations. Server vacation models are much useful in optimizing the production and the cost involved in any manufacturing system.

Analysis of some bulk queueing models with repair of service station, controllable services, state dependent arrivals, multiple vacations of different type with N-policy, multiple vacations of alternate type and multiple vacations with exceptional first vacation and exceptional last vacation with N-policy are presented in this dissertation.

The first chapter of this dissertation deals with the basics of queueing theory. A brief survey of some relevant existing research work on queueing models, viz., bulk queueing models with repair of service station on request, controllable services, state dependent arrivals and multiple vacations is presented.

Chapter two deals with the analysis of a **$M^x/G(a,b)/1$ queueing system with repair of service station on request and multiple vacations with exceptional last vacation**. The leaving batch of customers may request for repair of service station with probability π and it is assumed that more than one request will never be made by the same batch. After the repair time or service completion without request for repair, if the queue length is less than 'a', then the server avails multiple vacation till the queue length reaches 'a'. After a vacation, if the server finds atleast 'a' customers waiting for service (say) ξ, then the server requires an exceptional last vacation R to start the service. After this exceptional last vacation R, the server serves a batch of $\min(\xi,b)$ customers, where $b \geq a$. Using supplementary variable technique, the probability generating function of the steady state queue size at an arbitrary time is obtained. Expressions for the expected length of the queue, expected length of busy and idle periods are derived. Expected waiting time in the queue is also obtained. A cost model of the queueing system is developed. Numerical illustration is presented.

Chapter three is devoted for the study of a **$M^x/G(a,b)/1$ queueing system with N–policy and different types of vacation.** At a service initiation epoch, if the number of customers waiting in the queue is less than 'a', then the server selects the i^{th} type of vacation with probability α_i (i=1,2,........,M), where $\alpha_1+\alpha_2+........\alpha_M=1$, till the queue length reaches $N(N\geq b\geq a)$. After a vacation, if the server finds at least N customers waiting for service, then the server starts the service with a batch of 'b' customers. After a service if the number of customers waiting for service is ξ ($\xi\geq a$), then the server serves a batch of min(ξ,b) customers, where $b\geq a$. Using supplementary variable technique, the probability generating function of the steady state queue size at an arbitrary epoch is obtained. Expressions for the expected queue length, expected length of busy and idle periods are derived. Expected waiting time in the queue is also obtained. A cost model of the queueing system is developed. Numerical illustration is presented.

Chapter four deals with the analysis of a **$M^x/G(a,b)/1$ queueing system with fast and slow service rates and multiple vacations with exceptional first vacation.** The service rate can be controlled depending upon the queue length. The server does the service with a faster rate or at a slower rate based on the queue length. At a service initiation epoch, if the number of customers waiting in the queue is greater than or equal to N(N>b), then the service is done with a faster rate, otherwise with a slower rate. After completing a service, if the queue length is less than 'a', then the server avails an exceptional first vacation V_1. After this exceptional first vacation V_1, if the queue length is still less than 'a', then the server avails multiple vacations (different from first vacation) till the queue length reaches 'a'. After a service or a vacation, if the server finds atleast 'a' customers waiting for service (say) ξ, then the server serves a batch of min(ξ,b) customers, where $b \geq a$. Using supplementary variable technique, the probability generating function of the steady state queue size at an arbitrary epoch is obtained. Expressions for the expected queue length, expected length of busy and idle periods are derived. Expected waiting time in the queue is also obtained. A cost model of the queueing system is developed. Numerical illustration is presented.

Chapter five is devoted for the study of a **M^x/G(a,b)/1 queueing system with state dependent arrival and multiple vacations of alternate type.** The arrivals occur in bulk with a rate λ, when the server is busy and with a rate λ_2, when the server is on vacation. The service starts only when there are atleast 'a' customers waiting for service. After a service, if the queue length is less then 'a', then the server leaves for a vacation of type V_1. After returning from a vacation of type V_1, if still the queue length is less than 'a', the server leaves for another vacation of type V_2. The server avails these vacations V_1 and V_2 alternately, till the server finds, on returning from a vacation atleast 'a' customers waiting for service. After a vacation, if the queue length ξ is atleast 'a', then the server serves a batch of $\min(\xi,b)$ customers, where $b \geq a$. Using supplementary variable technique, the probability generating function of the steady state queue size at an arbitrary epoch is obtained. Expressions for expected queue length, expected length of busy and idle periods are derived. Expected waiting time in the queue is also obtained. A cost model of the queueing system is developed. Numerical illustration is presented.

Chapter six is concerned with the study of a **$M^x/G(a,b)/1$ queueing system with 2b–policy and multiple vacations with exceptional last vacation.** After completing a service, if the queue length ξ is less than 'a', then the server leaves for multiple vacation till the queue length reaches 'b'. After a vacation if the queue length ξ is atleast 'b', then the server avails an exceptional last vacation R. After this last vacation R, if the queue length ξ

is greater than or equal to '2b', then the server starts the service with a batch of 'b' customers, where $b \geq a$. Otherwise, the server remains in the system till the queue length reaches '2b'(by this assumption, the server has to complete atleast two batches of service before availing a vacation). The period in which the server is available in the system without serving the customer is called as the dormant period. The subsequent services are done with 'b' customers. After a service if the queue length ξ is such that $a \leq \xi \leq b$, then the server serves a batch of $\min(\xi,b)$ customers, where $b \geq a$. Using supplementary variable technique, the probability generating function of the steady state queue size at an arbitrary epoch is obtained. Expressions for expected queue length, expected length of busy and idle periods are derived. Expected waiting time in the queue is also obtained. A cost model of the queueing system is developed. Numerical illustration is presented.

Thus the present work is devoted to the analysis of some bulk queueing models with repair of service station on request, controllable services, state dependent arrivals, multiple vacations of different type with N-policy, multiple vacations of alternate type and multiple vacations with exceptional first vacation and exceptional last vacation with N- policy.