SOLVABILITY AND CONTROLLABILITY PROPERTIES OF STOCHASTIC DIFFERENTIAL EQUATIONS USING SEMIGROUP THEORY

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Abstract

Controllability plays an important role in stability theory and engineering that studies the influencing behavior of dynamical systems. Control theory aims to manipulate the dynamical system's behavior by introducing a control parameter. The dynamical systems encountered in engineering, physics, and economics often involve stochastic components and random noises, leading to models represented by stochastic differential equations. The real-world problems contain memory effects which can be described using fractional derivatives with non-singular kernels. The formulation of stochastic partial differential equations as stochastic ordinary differential equations in Hilbert space becomes crucial, as it highlights invariant properties while suppressing unnecessary details of individual problems.

This thesis investigates solvability, various types of controllability, and the existence of optimal control for stochastic differential equations in infinite-dimensional spaces. The Fredholm theory approach establishes the approximate controllability of semilinear retarded stochastic differential equations with non-instantaneous impulses driven by Poisson jumps. The optimal control of the second-order McKean-Vlasov stochastic evolution system with Clarke's subdifferential and mixed fractional Brownian motion is also discussed.

Furthermore, the thesis explores the solvability of the time-fractional stochastic Navier-Stokes equation and the optimal control of the time-fractional stochastic Burgers' equation, stochastic magnetohydrodynamic equations, and stochastic Gurtin-Pipkin integro-differential equations in Hilbert space. The trajectory controllability of fractional stochastic differential equations with Poisson jumps is also analyzed.

Several examples are provided throughout this work to make our theoretical analysis more concrete. The main techniques employed in this research include fixed point theorems, semigroup theory, fractional calculus, multi-valued analysis, and stochastic analysis theory.