BULK QUEUEING MODELS WITH N-POLICY, TWO SERVICE MODES, CLOSEDOWN TIMES AND MULTIPLE VACATIONS

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Queueing theory is concerned with the development of mathematical models to find the behaviour of a system that provides service for randomly arising demands. Since demands for service are assigned to be governed by some probability law, the theory of queues has been developed within the framework of the theory of stochastic processes.

The overwhelming majority of existing results in queueing theory pertain to systems that can provide services singly (one customer at a time). However, in many real life situations, service to the customers can be provided in bulk of different sizes. Also, there are situations in which customers arrive in bulk. In the queueing theory literature, such systems are investigated only in a lesser extent, may be due to inherent complexity. The idle time of the server may be effectively utilized by assigning other works (vacations). The server will be doing the secondary job when sufficient number of customers is not available to start a service. The objective of this dissertation is to analyse some bulk queueing models with multiple vacations and to discuss the application of the proposed models. The thesis is organized as follows:

Chapter one of the dissertation deals with the fundamental aspects of queueing theory. A literature survey of the relevant existing research work on queueing models, viz, bulk queueing models with vacations, two service modes and optional reservice is given. All the models discussed in this dissertation are solved using the supplementary variable technique.

A $M^X/G(a, b)/1$ **queueing system with multiple vacations and closedown times** is analysed in Chapter two. It is assumed that, after completing a service, if the queue length is less than 'a', the server performs closedown work (such as arranging the finished products). After completing the closedown work, the server leaves for a vacation of random length, irrespective of the queue length. After a vacation, if the queue length is still less than 'a', the server leaves for another vacation and so on, until he finally finds atleast 'a' customers waiting for service. After a vacation, if the server finds atleast 'a' customers waiting for service, say ξ, then he serves a batch of min (ξ, b) customers, where $b \geq a$. The probability generating function of the steady state queue size at an arbitrary time is obtained. Expressions for expected queue size, idle period, busy period and waiting time are also obtained. A cost model is discussed. A Numerical illustration is provided.

In Chapter three, a **Non-Markovian bulk arrival general bulk service queueing system with multiple vacations, setup time with N policy and closedown times** is considered. It is assumed that, on completion of a service, if the queue length is ξ, where $\xi < a$, then the server performs closedown work. After the closedown work, the server leaves for a vacation of random length irrespective of the queue length. After a vacation, if the queue length is less than 'N' ($N > b$), the server leaves for another vacation and so on, until he finally finds 'N' customers waiting for service. In the literature, queueing models considering this aspect is named as N-policy models. By this assumption of N-policy, the server will have to serve continuously atleast some batches, so that the operating cost will be minimized. After a vacation, if the server finds atleast 'N' customers waiting for service, then he requires a setup time R to start the service. After a setup time or on service completion, if the server finds ξ customers waiting for service, then he serves a batch of min (ξ, b) customers. The probability generating function of the steady state queue size at an arbitrary time is obtained. Expressions for expected queue size, idle period, busy period and waiting time are also obtained. A cost model is discussed. A numerical illustration is provided.

Chapter four deals with the **Analysis of two different $M^X/G/1$ queueing models by considering two service modes.** In the case of the first model, the service is rendered singly or in bulk, according to the queue length. The server starts the service, only if the number of customers in the queue is atleast 'a'. If the queue length is ξ, where $a \leq \xi < c$, then the server does single service till queue length reaches 'c' $(c > a)$, then the server switched over to bulk service. The bulk service is rendered with min (ξ, b) of customers, where $b > c$. The server switches over from single service to bulk service or vice-versa, only at service initiation epochs depending on the queue length. In the second model, the server selects the service mode as follows: if the server finds atleast one customer in the queue, then he starts single service till the queue length reaches 'a'. (In model 1, the server will wait in the system until the queue length reaches 'a', then he starts single service.) If the queue length is $\xi(\xi \geq a)$, then the server serves a batch of min (ξ, b) customers, where $b > a$. The server switches over from single service to bulk service or vice-versa, only at service initiation epochs, depending on the queue length. After a service, if the queue length is zero, the server avails a vacation of random length. After a vacation, if there are no customers waiting for service, then the server avails another vacation and so on, until he finds atleast one customer waiting in the queue. The probability generating function of the steady state queue size at an arbitrary time is obtained for the above two models. Also, the expressions for expected queue size, idle period, busy period and waiting time are obtained. A cost model is discussed. A numerical illustration is provided.

Chapter five is devoted to the study of a **Non-Markovian bulk arrival, general bulk service queueing system with multiple vacations and restricted optional reservice.** After the completion of an essential service, the leaving batch of customers may request for an reservice with probability π. The reservice is entertained only when the number of customers waiting in the queue is less than 'a'. After a reservice or after a service completion without request for reservice, if the queue length is less than 'a', then the server avails multiple vacation till the queue length reaches 'a'. At the completion epochs of vacation or essential service or reservice, if the queue length is ξ ($\xi \geq a$), then the server serves a batch of min (ξ, b) customers where b > a. The probability generating function of the steady state queue size at an arbitrary time is obtained. Expressions for expected queue size, idle period, busy period and waiting time are also obtained. A cost model is discussed. A numerical illustration is provided.

In Chapter six a **$M^X/G(a, b)/1$ queueing system with restricted vacations is analyzed.** After a service completion, if the number of customers waiting in the queue is less than 'a', then the server avails a vacation of random length. After a vacation, if the queue length is less than 'a', then the server avails another vacation and so on, until he finally finds atleast 'a' customers in the queue or he completes M number of vacations consecutively. After completing M^{th} vacation, if the queue length is still less than 'a', then the server remains in the system till the queue length reaches 'a' (this period is known as dormant period). At a vacation completion epoch or service completion epoch or dormant period completion, if the queue length ξ is atleast 'a', then the server serves a batch of min (ξ, b) customers, where $b > a$. The probability generating function of the steady state queue size at an arbitrary time is obtained. Expressions for expected queue size, idle period, busy period and waiting time are also obtained. A cost model is discussed. A numerical illustration is provided.

In the last Chapter, a live production line problem is discussed to illustrate how the results obtained influence the production line system. Various tables are presented and the results are illustrated graphically to show, how the management can use these results to optimize the cost function. Specific parameters that can optimize the cost functions of the respective models are determined numerically.

Thus, the present work is devoted to the analysis of some bulk queueing models with two service modes, optional reservice, multiple vacations and restricted number of vacations.