Finite element methods (FEMs) are a popular simulation technique for discretizing partial differential equations (PDEs) to an algebraic problem. For well-studied problems, theory shows which methods are optimal and can be applied to a wide variety of applications. The accuracy of numerical simulation algorithms is one of main concerns in modern Computational Fluid Dynamics. Development of new and more accurate mathematical models requires an insight into the problem of numerical errors. In this thesis, we adopted a finite element method to compute incompressible fluid flows and heat transfer. In particular, we first tested the method, implemented in FEniCS software library for Cahn-Hilliard equation, then we computed 2D, fully coupled, time-dependent incompressible fluid flow and heat transfer poblems. In this thesis, we start our work with the study of existence and uniqueness of approximate solution for a time-discretized Navier-Stokes equations with a slip-like boundary conditions. Next, we study the laminar fluid flow and heat transfer over a backward facing step with an adiabatic rotating cylinder for various cross-stream position, without cylinder and rotating heated cylinder. In the next chapters, numerical analysis of the heat transfer enhancement and fluid flow characteristics of a stationary and rotating cylinder in a T-channel are performed. Used obstacle in flow passage leads to increase the static pressure and then enhance the heat transfer. Furthermore, length and size of the flow separation, recirculation and reattachments can be controlled with cylinder rotation angle.