ABSTRACT

The concept of spectrum of an operator is a generalization of the concept of eigen values for matrices. Spectral theory or Spectral analysis of linear operators is one of the most dynamic area in operator theory. In 1909, writing about differential equations, Hermann Weyl noticed something about the essential spectrum of a self - adjoint operator on Hilbert space : when you take it away from the spectrum, you are left with the isolated eigen values of finite multipicity. This was soon generalized to normal operators, and then to more and more classes of operators, bounded and unbounded, on Hilbert and on Banach spaces. He examined the spectra of all compact perturbations $T + K$ of a single hermitian operator T and discovered that $\lambda \in \sigma(T+K)$ for every compact operator K if and only if λ is not an isolated eigenvalues of finite multiplicity in $\sigma(T)$, where $\sigma(T+K)$ is the spectrum of $T+K$ and $\sigma(T)$ is the spectrum of T. This remarkable result is known as Weyl's theorem.

Let $\mathcal{H}$ be a Hilbert space and T be a bounded linear operator defined on $\mathcal{H}$. An operator T is said to be absolute - (p, r) - paranormal operator if $\||T|^p|T^*|^r x\|^r \geq \||T^*|^r x\|^{p+r}$ for every unit vector x or equivalently $\||T|^p|T^*|^r x\|^r\|x\| \geq \||T^*|^r x\|^{p+r}$ for all $x \in \mathcal{H}$ and for positive real numbers $p > 0$ and $r > 0$, where $T = U|T|$ is the polar decomposition

of T. In chapter 2, we prove that Weyl's theorem holds for invertible absolute - (p, r) - paranormal operators and present several applications of Weyl's theorem. We prove the necessary and sufficient condition for self - adjointness of Riesz projection associated with an isolated point of spectrum of invertible absolute - (p, r) - paranormal operator.

An operator $T \in B(\mathcal{H})$ is called algebraically absolute - (p, r) - paranormal if there exists a non constant polynomial p such that $p(T)$ is absolute - (p, r) - paranormal. In chapter 3, Weyl's theorem for algebraically absolute - (p, r) - paranormal operators is studied.

In chapter 4, we prove the continuity of the set theoretic functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum on the classes consisting of (p, k) - quasihyponormal operators and invertible absolute - (p, r) - paranormal operators. We prove that if $\{T_n\}$ is a sequence of operators in the class (p, k) - quasihyponormal or invertible absolute - (p, r) - paranormal which converges in the operator norm topology to an operator T in the same class, then the functions spectrum, Weyl spectrum, Browder spectrum and essential surjectivity spectrum are continuous at T.

In chapter 5, absolute - (p, r) - paranormal composition operators and p - paranormal composition operators on L^2 space are characterized. Also we characterize weighted absolute - (p, r) - paranormal and weighted p - paranormal composition operators.