MODELING AND EXPERIMENTAL INVESTIGATION OF EFFECTIVE THERMAL CONDUCTIVITY OF HOMOGENEOUS AND HETEROGENEOUS TWO-PHASE MATERIALS

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P. KARTHIKEYAN

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DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY MADRAS

CHENNAI-600 036, INDIA

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ABSTRACT

KEYWORDS: Effective Thermal Conductivity; Two-phase system; Unit-Cell model; Isotherms; Field solution model; Collocated parameter model; Foods; Square Guarded Hot Plate.

The problem of finding out the effective thermal conductivity of two-phase system has been a classical one; but has still defied exact solution. The essential ingredient, which makes it theoretically an unsolvable problem, is the fact that it is a confluence of many complicated geometrical and structural problems. The exact solution of the problem is solving the Laplace equation for the field of many structures, and it has not been possible to obtain an exact solution. The only successful attempt to obtain a field solution is by James Clark Maxwell (1873) for predicting magnetic permittivity. Maxwell solution is applicable for vanishingly small concentration of the dispersed phase. After that there has been lot of research in this field, both theoretical and experimental, because of its growing importance in the industry.

The estimation of effective thermal conductivity is influenced by primary parameters such as concentration (υ) and conductivity ratio (α). In addition to the primary parameters there are the secondary effects and geometrical parameters influencing the effective conductivity. At present, there is no satisfactory solution for all ranges of conductivity ratio and concentration, because the neighbouring interactions on the field produce higher order effects, which are difficult to model. The objective of the present work is to propose the possible approaches and solutions to the classical problem. A novel approach is proposed to predict the effective thermal conductivity of two-phase materials based on unit cell approach including higher order interactions and secondary effects (parallel isotherms) as well as on the semi-empirical field solution approach valid for full ranges of concentration ($0 < \upsilon < 1$) and conductivity ratio ($0 < \alpha < \infty$).

The unit cell model is proposed for the estimation of thermal conductivity of two-phase materials having $0.1 \leq \upsilon \leq 0.9$ and low conductivity ratio ($\alpha < 20$). The field solution model is proposed to predict the effective conductivity of two-phase systems incorporating the primary parameter. For $\alpha \geq 20$, the field solution model is proposed for three α ranges viz. medium ($20 \leq \alpha \leq 100$), high ($100 \leq \alpha \leq 1000$) and very high ($1000 > \alpha$). For any α, Maxwell and Phase inverted Maxwell equations are considered respectively for $0 \leq \upsilon < 0.1$ and $0.9 < \upsilon \leq 1$.

In the collocated parameter model, the algebraic equations are derived using unit cell based parallel isotherms for two and three-dimensional spatially periodic two-phase medium involving higher order interactions as well as secondary effects. The geometry of the medium is considered as a matrix of touching and non-touching in-line square and circular cylinders as well as touching and non-touching in-line solid and hollow cubes. The models are used to predict the thermal conductivity of numerous two-phase materials in the full range of concentration and conductivity ratios ($0 < \alpha < 100$ and $0 < \upsilon < 1$). The non-dimensional thermal conductivity of two-phase materials is influenced by concentration, conductivity and contact ratios, and height of inclusion in the unit cell. The model has been validated with published experimental data for various two-phase systems. The predicted effective thermal conductivity is more accurate than the earlier models. The two-dimensional spatially periodic in-line touching square cylinder model has been combined with four fundamental models to evolve a unifying equation for estimation of effective conductivity of heterogeneous two-phase materials such as frozen and un-frozen food materials.

With a view to validate the models, a steady state thermal conductivity measurement system, Square Guarded hot plate (SGHP) apparatus is designed. The thermal modeling of SGHP apparatus and its components such as the meter plate, coolant block, and edge guard is carried out to evolve an optimum configuration. The collocated parameter model is validated with thermal conductivity of high-density insulation materials. The results are in good agreement with the analytical model with in the maximum deviation of ± 21 % for heavy insulating plates. Based on the thermal modeling results, a full-scale square guarded hot plate setup is fabricated to measure the thermal conductivity of insulation materials in the range of 0.035 to 2 W/m K with a combined measurement uncertainty of ± 5.75 % at maximum operating temperature of 623 K. The SGHP experimental set up is calibrated with Standard Reference Material (SRM) procured from National Physical Laboratory, United Kingdom.