

## ABSTRACT

The objective of the thesis is to investigate the performance of constrained linear quadratic optimal control for stabilization and tracking applications of under actuated system. Under actuated systems are mechanical control systems which have fewer independent actuators than the degrees of freedom to be controlled. Due to their broad applications in robotics, locomotive systems and aircraft, under actuated systems constitute a good framework of nonlinear control problems of both theoretical and practical interests.

In this thesis, we evaluate the performance of the constrained linear quadratic optimal controller on three bench mark under actuated systems namely inverted pendulum, magnetic levitation and torsional system. One of the major challenges in the design of linear quadratic regulator (LQR) for under actuated systems is the choice of weighting matrices which will result in optimal response. The weighting matrices  $Q$  and  $R$  regulate the penalties on the excursion in the trajectories of the state variables and control input respectively. With random choice of  $Q$  and  $R$  matrices, the optimal regulators do not provide good set point tracking response because of the absence of integral term. The weighting matrices in the cost function are normally chosen based on trial and error approach to determine the state feedback gain, but this approach is not only tedious but also time consuming. Moreover, manual selection of weighting matrices may not result in optimal response. Hence, to address the weight selection problem of LQR, we propose an Adaptive Particle Swarm Optimization (APSO) algorithm for selecting the  $Q$  and  $R$  matrices of LQR. An adaptive PSO to improve the convergence rate of conventional PSO is employed for tuning the gains of LQR. One of the notable changes of the proposed APSO is that the weights in the velocity

update equation of conventional PSO are adjusted adaptively in accordance with the success rate of the particles towards the best value. An adaptive inertia weight factor (AIWF) is adjusted adaptively not only to speed up the search process but also to increase the accuracy of the algorithm. Stabilization of inverted pendulum is formulated as LQR optimization problem, and the selection of Q and R matrices is solved using the adaptive PSO. The efficacy of the APSO in providing the improved convergence results is tested on a Quanser inverted pendulum model, and the performance of the proposed APSO is compared with that of the conventional PSO and GA.

In addition, an algebraic weight selection algorithm based on the Algebraic Riccati Equation (ARE) is formulated to choose the elements of weighting matrices in accordance with the design criteria represented in time domain. The key idea of this method is that the solution of ARE is used to integrate the time domain specifications of the system with its canonical state space model, so that the coefficients of weighting matrices can be obtained as a direct function of the design requirement, which makes the selection of weighting matrices straight forward and avoids the tedious task of manual tuning. There are two advantages of this algebraic approach over iterative manual tuning. First, by relating the coefficients of weighting matrices in terms of design specification, it significantly reduces the time needed for the design of LQR. Second, the design framework translates the performance requirement of the system in time domain into cost function, which makes the design simple and modular. The effectiveness of the proposed approach is validated on a fourth order torsional system for tracking applications.

Motivated by one of the typical behaviours of LQR, minimizing the excursion in the state trajectories of a system by maintaining minimum control effort, we integrated the tuning philosophy of PID with the control concepts of LQR to facilitate both the optimal set point tracking and optimal

cost of control within the same design framework. The LQR based PID controller strategy is evaluated on a bench mark magnetic levitation system for tracking applications. The effectiveness of the controller framework to track the various test signals are validated in real time.