ABSTRACT

Solving system of linear algebraic equations, determining the roots of a real algebraic polynomial and evaluating determinant value of a matrix are ancient problems in engineering applications, however there is an increasing demand for computationally stable and efficient numerical algorithms converging to the actual results of these problems. The proposed research provides novel techniques to enhance convergence rate of the numerical algorithms to solve these engineering problems in terms of accuracy and computational time.

The condition of these conventional mathematical problems relates to their sensitivity to the input perturbation. A computation is numerically unstable if the uncertainty of the input values is grossly magnified by the numerical method. When the results of numerical computations are extremely critical and cause loss of human life or have severe economic or social implication, it is appropriate to take special precautions. Therefore, handling Ill-conditioning of these problems and applying soft computing technique for optimization have been incorporated in the proposed algorithms.

The iterative numerical algorithms for root extraction from an algebraic polynomial are based on an initial approximation of one or

two roots. The convergence on the result depends mainly on the nearness of the approximate root to the actual one. The problem of convergence on inaccurate results or divergence from actual roots due to improper initial approximation which is applied to numerical algorithms is addressed in this thesis.

In this thesis, the design of numerical algorithms to determine the roots of an algebraic polynomial aims at two objectives: i) faster convergence and ii) actual results with the specified accuracy. Objective one, the faster convergence is achieved by formulating a technique to extract the initial approximant from the given polynomial, instead of guessing an initial approximant. This extracted initial approximant must be closely located to actual roots. Objective two, the actual results are obtained by developing an error correction technique applied to the approximant which follows the characteristic of the roots of the polynomial. This technique ensures the successive iterations converge to the actual roots. To achieve these two objectives, Triangular method and Extended Routh-Hurwitz Criterion methods are proposed in this thesis. The application of the proposed algorithms to extract the roots of different class of polynomials is the central theme of this research work.

This thesis also handles the major issue in root-finding numerical technique for algebraic polynomial which is sensitive to coefficient perturbations. The location of roots is sensitive to perturbations. The polynomials in which small changes in the coefficients can cause large displacement of roots are considered to be ill-conditioned polynomials. For such class of polynomials, optimal solution is proposed. The incremental search for roots is optimized using the tools in the AFSA optimization techniques.

The numerical solution techniques either direct or iterative for solving system of linear algebraic equations should deal the following drawbacks in their procedures: division by zero, round-off errors, ill-conditioned systems and singular systems. The outcome of the research work carried on these drawbacks results in development of new numerical techniques in this thesis for solving the system of linear equations and evaluating the determinant value of the linear system.

The present technique for solving the linear system follows the direction of direct methods but involves the simple procedure for forward reduction without using division operation. Therefore, pivoting technique which induces round-off error can be avoided. Also, a simple scheme for handling ill-conditioned system is also incorporated in this proposed algorithm.

The most of the common methods evaluate the determinant by multiplying the manipulated diagonal elements. These diagonal elements are obtained by reduction process. If any of the diagonal elements is zero then further reduction is not possible. Also if any diagonal element of the determinant is small, then the division leads to the loss of significant digits. To prevent the loss of significant digits, the pivoting techniques are used. To overcome this drawback, a numerical technique is proposed in this thesis, which converts the given determinant into a well-conditioned one by applying a simple procedure. Thus, pivot technique can be avoided at every stage of the procedure. Then, the determinant is evaluated by extracting the column vectors and multiplying them all.

The appropriate numerical algorithms are developed in this thesis for the proposed techniques. These algorithms are simple in application and can be implemented using any programming languages. The illustrative problems tested on these algorithms against common methods show the performance efficiency of the proposed algorithms.